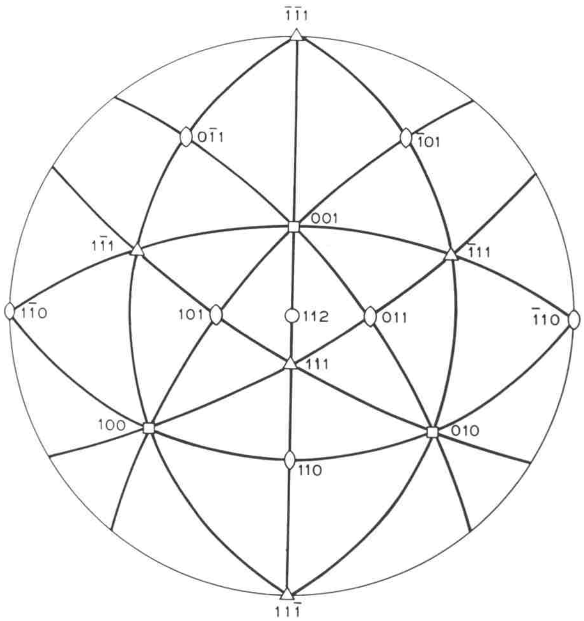

FIG. 1. Standard (112) stereographic projection for a cubic crystal.

Given a crystal orientation, the strain components along the cubic axes ($d\varepsilon_{11}$, etc.) can be found in terms of $d\varepsilon_{xx}$ by using equation (6) and the proper matrix for transforming coordinate axes. The appropriate combination(s) of A, B, etc. can be then be found from among the 28 stress states to maximize the right side of equation (5).

APPLICATION TO F.C.C. METALS

Equation (5) will now be applied to crystals of several highly symmetrical orientations. Because of their symmetry, these orientations are of interest in connection with texture formation and magnetic anisotropy[8] as well as strength considerations. As usual, $\{111\}\langle 110\rangle$ slip is assumed.

1. Compression plane (112); elongation direction $[\bar{1}\bar{1}1]$.

Let the specimen coordinate axes be x—[112], y—[110], and z—$[\bar{1}\bar{1}1]$, Fig. 1. The matrix of transformation to the cubic axes (1—[100], 2—[010], 3—[001]) is

	x	y	z
1	$\frac{1}{\sqrt{6}}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{3}}$
2	$\frac{1}{\sqrt{6}}$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{3}}$
3	$\frac{2}{\sqrt{6}}$	0	$\frac{1}{\sqrt{3}}$

Hence, from equation (6) and the transformation matrix, we find that

$$d\varepsilon_{11} = -d\varepsilon_{xx}/6, \quad d\varepsilon_{22} = -d\varepsilon_{xx}/6, \quad d\varepsilon_{33} = d\varepsilon_{xx}/3$$
$$d\varepsilon_{23} = 2d\varepsilon_{xx}/3, \quad d\varepsilon_{31} = 2d\varepsilon_{xx}/3, \quad d\varepsilon_{12} = -d\varepsilon_{xx}/6, \tag{7}$$

and equation (5) becomes

$$\begin{aligned} M &= \frac{1}{\tau}\left[\frac{B}{6} - \frac{A}{6} + \frac{4}{3}F + \frac{4}{3}G - \frac{H}{3}\right] \\ &= \frac{1}{6\tau}[B - A + 8F + 8G - 2H] \end{aligned} \tag{8}$$

By trial, the Bishop and Hill stress states that maximize the right side of equation (8) are Nos. 21 and 25 in Table 1. For No. 21, $A = -\frac{1}{2}$, $B = \frac{1}{2}$, $C = 0$, $F = G = \frac{1}{2}$, $H = 0$; and for No. 25, $A = B = C = 0$, $F = G = \frac{1}{2}$, $H = -\frac{1}{2}$, all multiplied by $\sqrt{6}\,\tau$. We thus have $M = 3\sqrt{6}/2$ in equation (8).

It may be noted that for stress state 21, the active slip systems are $-a_1$, a_2, c_2, $-c_3$, $-d_1$ and d_3 (Bishop and Hill notation, see Appendix 1), while for stress state 25, they are $-a_1$, a_2, c_1, $-c_3$, $-d_2$ and d_3. Since the deformation borders on both stress states, only those slip systems (with proper sign) common to both are activated. This means $-a_1$, a_2, $-c_3$ and d_3.

2. Compression plane (110); elongation direction $[\bar{1}12]$.

Let the specimen axes be x — [110], y — $[\bar{1}1\bar{1}]$ and

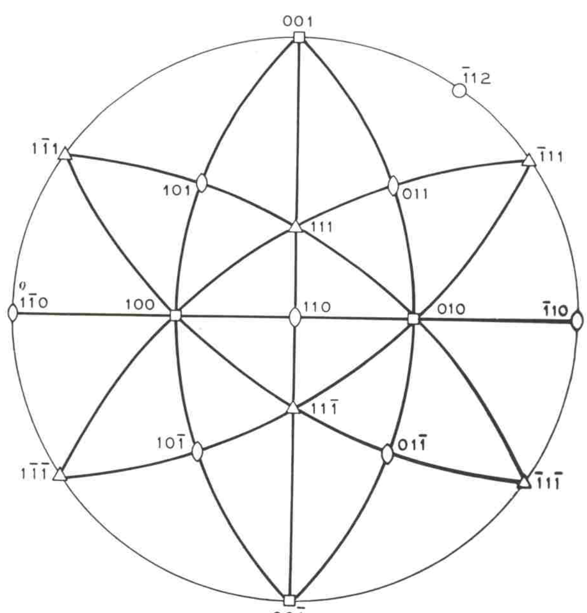

FIG. 2. Standard (110) stereographic projection for a cubic crystal.

z — $[\bar{1}12]$, Fig. 2. The matrix for transformation to the cubic axes is

	x	y	z
1	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{6}}$
2	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{6}}$
3	0	$-\frac{1}{\sqrt{3}}$	$\frac{2}{\sqrt{6}}$

From equation (6) and the transformation matrix, we obtain

$$d\varepsilon_{11} = d\varepsilon_{22} = d\varepsilon_{xx}/3,\ d\varepsilon_{33} = -2d\varepsilon_{xx}/3,$$
$$d\varepsilon_{23} = -d\varepsilon_{xx}/3,\ d\varepsilon_{31} = d\varepsilon_{xx}/3,\ d\varepsilon_{12} = 2d\varepsilon_{xx}/3. \quad (9)$$

Equation (5) becomes

$$M = \frac{1}{\tau}\left[-\frac{B}{3}+\frac{A}{3}-\frac{2F}{3}+\frac{2G}{3}+\frac{4H}{3}\right] = \frac{1}{3\tau}[-B+A-2F+2G+4H]\,. \quad (10)$$

The Bishop and Hill stress states that maximize the right side of (10) are Nos. 6 and 27, with $M = 4\sqrt{6}/3$.

For stress state 6 ($A = B = C = F = G = 0, H = \sqrt{6}\,\tau$), slip systems a_1, $-a_2$, b_1, $-b_2$, $-c_1$, c_2, $-d_1$, d_2 become active. And for state 27 ($A = B = C = 0$, $F = -\sqrt{6}\,\tau/2$, $G = H = \sqrt{6}\,\tau/2$), $-a_2$, a_3, b_1, $-b_3$, $-d_1$ and d_2 become active. Hence, the actual operating systems (which are common to both states) are $-a_2$, b_1, $-d_1$ and d_2. It may be noted that systems $-d_1$ and d_2 are in cross-slip relationship with b_1 and $-a_2$, respectively.

A closer examination of this orientation, however, reveals the possibility of slip on systems $-a_2$ and b_1 alone. It may be shown that under these conditions,

$$d\varepsilon_{xx} = -\frac{2}{\sqrt{6}}\,d\gamma,\quad d\varepsilon_{yy} = 0,\quad d\varepsilon_{zz} = \frac{2}{\sqrt{6}}\,d\gamma,$$
$$d\varepsilon_{yz} = -\frac{\sqrt{3}}{6}\,d\gamma,\quad d\varepsilon_{zx} = 0,\quad d\varepsilon_{xy} = 0, \quad (11)$$

where $d\gamma$ is the incremental shear each on slip systems $-a_2$ and b_1. The only difference between equations (11) and (6) is in the shear strain term $d\varepsilon_{yz}$. However, since the present setup does not restrict $d\varepsilon_{yz}$ to zero, the deformation is expected to occur on $-a_2$ and b_1 alone *if* the total amount of shear $\Sigma|d\gamma_i|$ is less than that for the four slip systems case (Taylor's minimum shear principle). For slip on $-a_2$ and b_1, we have from equation (11),

$$\Sigma\,|d\gamma_i| = 2\,|d\gamma| = \sqrt{6}\,d\varepsilon_{xx}, \quad (12)$$

and for slip on $-a_2$, b_1, $-d_1$ and d_2,

$$\Sigma\,|\,d\gamma_i\,| = Md\varepsilon_{xx} = \frac{4}{3}\sqrt{6}\,d\varepsilon_{xx}. \quad (13)$$

The former value is one-third less and hence we may expect slip on $-a_2$ and b_1 alone.

Similar calculations were carried out for five other orientations of interest. The results for all seven orientations are summarized in Table 2, together with those for the polycrystalline samples. Interestingly, the same operating slip systems as those listed in Table 2 were found earlier by a less rigorous method.[8] As for the polycrystalline material, a value of $M = 1.44\sqrt{6}$ was used. It was derived by Hosford and Backofen[3] from the von Mises yield criterion and the use of Taylor's factor of 3.06 for relating the tensile yield stress to the resolved shear stress for slip in a randomly oriented polycrystalline sample. Although the derivation was based on tensile testing under plane strain conditions, it may be shown that this value is

TABLE 2. Summary of analysis

Sample no.	Compression plane	Elongation direction	M	Slip systems selected	Equation (6) satisfied?
1	112	$\bar{1}\bar{1}1$	$3\sqrt{6}/2$	$-a_1, a_2, -c_3, d_3$	yes
2	110	$\bar{1}12$	$\sqrt{6}$	$-a_2, b_1$	no
			$4\sqrt{6}/3$	$-a_2, b_1, -d_1, d_2$	yes
3	110	001	$\sqrt{6}$	$a_1, -a_2, b_1, -b_2$	yes
4	110	$\bar{1}10$	$2\sqrt{6}$	$-a_1, a_2, -b_1, b_2, c_1, -c_2, d_1, -d_2$	yes
5	001	$\bar{1}00$	$\sqrt{6}$	a_2, b_2, c_2, d_2	yes
6	001	$\bar{1}10$	$\sqrt{6}$	$-c_1, c_2, -d_1, d_2$	yes
7	111	$\bar{1}\bar{1}2$	$3\sqrt{6}/2$	$-b_1, b_2, c_3, -d_3$	yes
8	polycrystal	#1	$1.44\sqrt{6}$	—	yes
9	polycrystal	#2	$1.44\sqrt{6}$	—	yes

Composition of all samples: 4% Mo– 17% Fe– 79% Ni by weight. Samples 8 and 9 were slowly cooled and quenched, respectively, after annealing at 1000°C; grain diameter ~0.04 mm.